

Influence of absorbing dielectric background on bistable response of dense collection of two-level atoms

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The stationary problem of light interaction with a dense collection of two-level atoms embedded in a dielectric is treated semiclassically. The effect of dielectric absorption on the phenomenon of intrinsic optical bistability is discussed in terms of the complex local-field enhancement factor. The calculations accounting propagation effects (nonlocal regime) show that even low dielectric absorption results in elimination of response instabilities.

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I. INTRODUCTION

The effect of optical bistability is the corner-stone of optical information processing. It is the basis for implementation of such devices as all-optical switches, transistors, and logical gates. The straightforward method to obtain the bistable response is to place a nonlinear medium inside a feedback system, for example, Fabry–Perot resonator [1]. In the late 1970s Bowden and Sung introduced [2] the idea of the mirrorless (intrinsic) optical bistability as a result of interatomic (dipole-dipole) correlations. This idea was then developed in a number of papers (see, for example, [3–5]).

The usual starting point in the analysis of intrinsic optical bistability (IOB) is the semiclassical generalized Bloch equations [6] which take into account the near-dipole-dipole interactions of two-level atoms (the so-called local field correction). However, there is a problem of influence of the background (host) dielectric medium on the bistable response. Crenshaw proved [7] that the semiclassical approach gives the correct explanation of local-field enhancement by the linear dielectric in accordance with the *microscopic* quantum electrodynamics (QED). Interestingly, this enhancement cannot be accounted for by the *macroscopic* QED. Therefore, if the process of spontaneous emission is not to be examined, we can use the semiclassical Bloch equations.

The local-field enhancement due to the host dielectric is measured by the factor $\ell = (\varepsilon + 2)/3$, where ε is the dielectric permittivity. Since ε can be complex, we can automatically take into account dielectric absorption. It was shown in Ref. [8] that the real part of ℓ reduce the IOB threshold while the imaginary part acts in the opposite direction. In one of our previous publications [9] we studied the effects of local and nonlocal bistability and internal coherent reflection under the assumption of absence of the background dielectric influence. In this short paper we try to fill this omission.

The paper is divided into several parts. Section II

is devoted to the solution of the stationary problem of monochromatic radiation interaction with the dense collection of two-level atoms in a dielectric. We analyze the influence of dielectric properties on the main parameters of local bistability. In Section III we consider the overall permittivity of such a media and its bistable response. Finally, in Section IV the problem of radiation propagation in a layer of finite thickness is discussed. The effect of dielectric on the so-called nonlocal bistability is numerically analyzed.

II. BISTABLE RESPONSE

In the paper we consider the normal incidence of linearly polarized monochromatic plane wave on a dense resonant medium. We start with semiclassical Bloch equations which take into account near dipole-dipole (NDD) interactions of two-level atoms as well as background dielectric effect [7, 8],

$$\frac{\partial R}{\partial t} = -i\frac{\mu}{2\hbar}\ell EW + i(\Delta - \ell\epsilon W)R - \gamma_2 R, \quad (1)$$

$$\begin{aligned} \frac{\partial W}{\partial t} = & -i\frac{\mu}{\hbar}(\ell^* E^* R - \ell ER^*) - 2i(\ell^* - \ell)\epsilon|R|^2 \\ & - \gamma_1(W - W_{\text{eq}}), \end{aligned} \quad (2)$$

where W is the inversion (population difference of two levels), R is the atomic polarization, E is the amplitude of macroscopic electric field, μ is the transition dipole moment, Δ is the detuning of radiation from resonance, γ_1 and γ_2 are the population and polarization relaxation rates respectively, W_{eq} is the value of inversion at equilibrium. The parameter

$$\epsilon = \frac{4\pi N\mu^2}{3\hbar} \quad (3)$$

is responsible for NDD interaction strength, N is the density of two-level atoms per unit volume, \hbar is the Planck constant. Here $\ell = (\varepsilon + 2)/3$ is the local-field enhancement factor due to polarizability of host material with dielectric constant ε (generally, complex). Appearance

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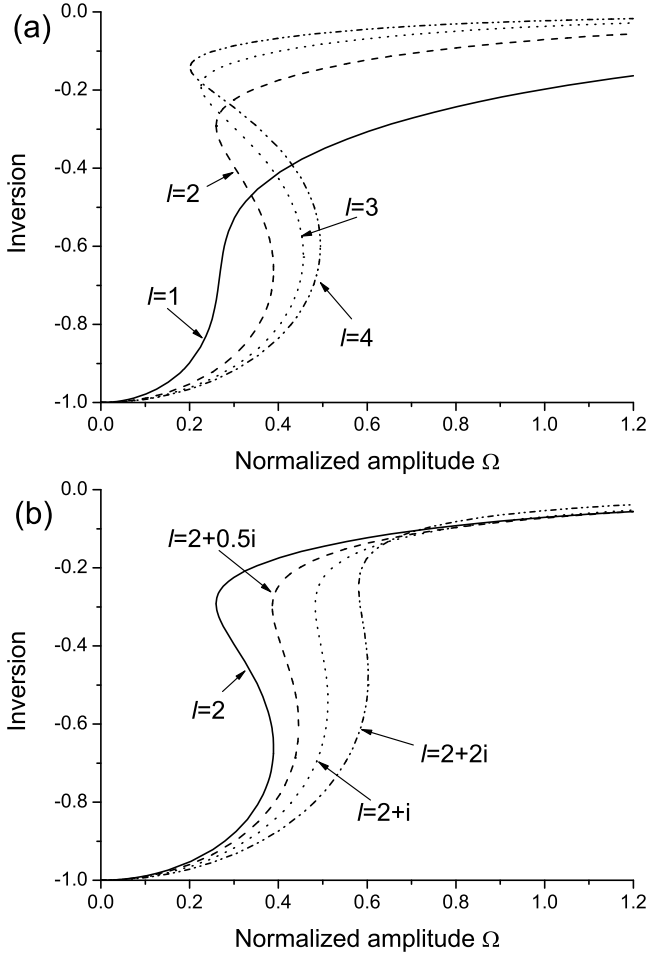


FIG. 1: Dependence of inversion on the light amplitude at (a) real values of factor ℓ , (b) complex values of factor ℓ . Calculation parameters: $W_{\text{eq}} = -1$, $\delta = -2$, $b = 4$, $\Gamma = 0.1$.

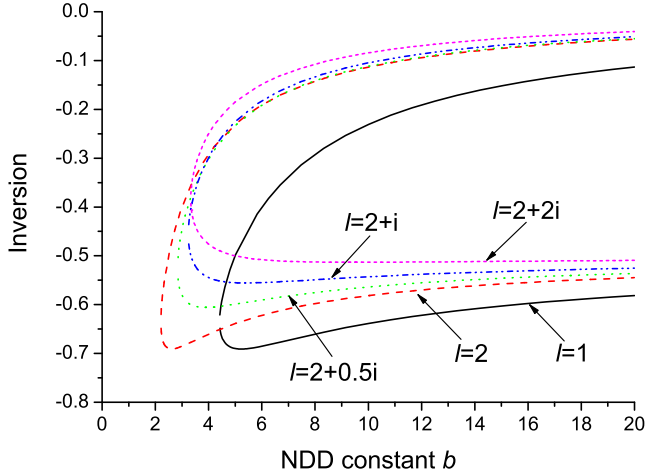


FIG. 2: (Color online) Positions of the inflection points as a function of the NDD parameter b . Other parameters: $W_{\text{eq}} = -1$, $\delta = -2$.

of factor ℓ results in three local-field effects [8]: (i) en-

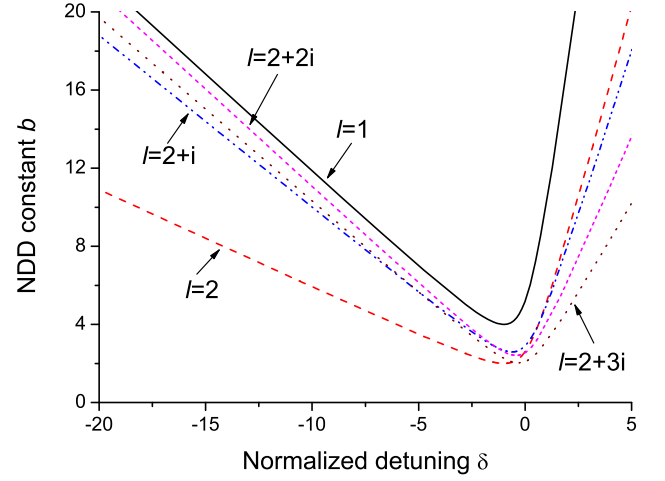


FIG. 3: (Color online) The region of bistability existence in the plane (δ, b) .

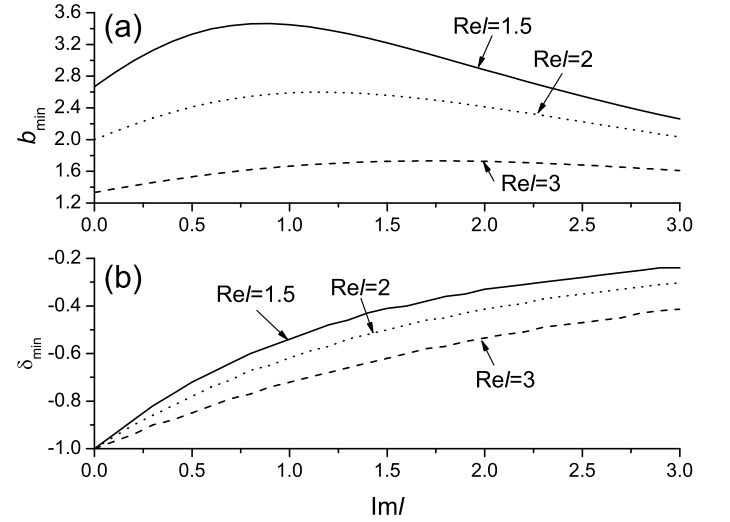


FIG. 4: The dependence of minimal NDD parameter value (a) and corresponding detuning (b) on the imaginary part of the local-field enhancement factor ℓ .

hancement of the magnitude and the phase shift (if ε is complex) of the electric field; (ii) Lorentz frequency shift ($\omega_L = \varepsilon W$) enhancement due to real part of ℓ ; (iii) cooperative decay due to imaginary part of ℓ in both equations (1) and (2).

In stationary regime we obtain the expression for the polarization, which can be written in dimensionless notation as follows

$$R = \frac{1}{2} \frac{i\ell W \Omega}{i(\delta - \ell b W) - 1}, \quad (4)$$

where $\Omega = \mu E / \hbar \gamma_2$, $\delta = \Delta / \gamma_2$, $b = \varepsilon / \gamma_2$. It is seen that the polarization depends on the inversion which can be calculated from the equation

$$\Gamma(W - W_{\text{eq}}) |i(\delta - \ell b W) - 1|^2 + W |\ell \Omega|^2 = 0. \quad (5)$$

Here $\Gamma = \gamma_1/\gamma_2$.

The cubic equation (5) describes the phenomenon of intrinsic optical bistability (IOB) well known from the previous researches. However, the presence of dielectric background changes the quantitative characteristics and conditions of IOB. As can be seen from Fig. 1(a), the Lorentz shift enhancement (the second effect named above) results in easier observation of bistable response. In vacuum case ($\ell = 1$), if the other parameters are the same, there is no bistability at all. On the other hand, if the dielectric is absorptive [complex factor ℓ , see Fig. 1(b)], bistable loop not only shifts towards higher light intensities, but also gets more and more narrow.

This implies that absorption of the background material changes the region of IOB existence somehow. Let us consider this question in detail. The necessary condition for the Eq. (5) to demonstrate bistability is that the equation $d|\Omega|^2/dW = 0$ has two different roots in the physically appropriate range $-1 \leq W \leq 0$. This equation can be written in explicit form as

$$2b^2|\ell|^2W^3 - b(2\text{Im}\ell + 2\delta\text{Re}\ell + b|\ell|^2W_{\text{eq}})W^2 + W_{\text{eq}}(\delta^2 + 1) = 0. \quad (6)$$

Two solutions of Eq. (6) in the range $-1 \leq W \leq 0$ represent the inflection points of bistability which are seen in the Fig. 1 (the points of jump from one solution to another). When these points coincide, the bistability loop disappears. Figure 2 shows the behavior of the inflection points as NDD parameter is decreasing. The critical (minimal) value of b at which bistability still exists decreases for larger real parts of the local-field enhancement factor and increases for larger imaginary parts of ℓ . At the same time the inversion corresponding to the jumps between the solutions is growing.

This means that the region of parameters δ and b , where the bistability, occurs changes as well. Some examples of these regions of bistability existence are demonstrated in Fig. 3: above the curves plotted the bistability is present. In the case of vacuum ($\ell = 1$) we obtain the curve that was reported previously [9]. The minimal value of the NDD parameter in this case is $b_{\text{min}} = 4$ for the detuning $\delta_{\text{min}} = -1$. If ℓ increases and is still real, the region of bistability is getting wider. For example, for $\ell = 2$ we have $b_{\text{min}} = 2$ at the same value of the detuning $\delta_{\text{min}} = -1$. However, if the background dielectric is absorptive (ℓ is complex), this minimal value of NDD parameter increases.

From Fig. 3 one can see that b_{min} for $\text{Im}\ell = 1$ is larger than for $\text{Im}\ell = 0$. However, for $\text{Im}\ell = 2$ it is already smaller than in the former case. Detailed study of the behavior of the minimal value of the NDD parameter [Fig. 4(a)] shows that there is a maximum in dependence $b_{\text{min}}(\text{Im}\ell)$. As $\text{Im}\ell$ tends to infinity, the b_{min} is slowly decreasing down to zero. At the same time the value of detuning, corresponding to the minimal NDD parameter, monotonically tends to zero, too. Note that, for smaller $\text{Re}\ell$, the curve $b_{\text{min}}(\text{Im}\ell)$ has more pronounced maximum

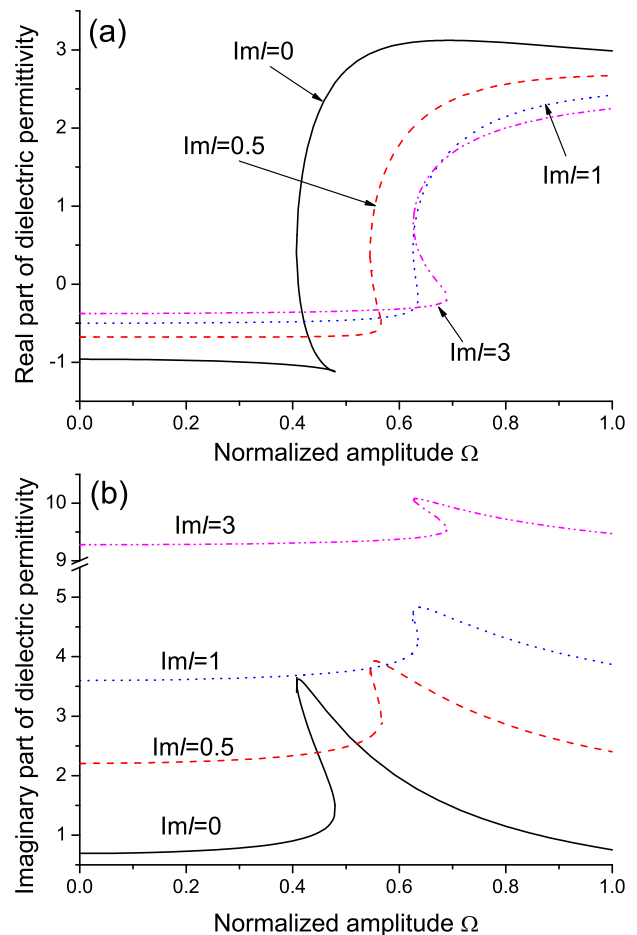


FIG. 5: Dependence of (a) real and (b) imaginary parts of the dielectric permittivity on the light amplitude. Calculation parameters: $W_{\text{eq}} = -1$, $\delta = -1$, $b = 4$, $\Gamma = 0.1$, $\text{Re}\ell = 1.5$, $\text{Im}\ell$ is a varying parameter.

and then slower converge to zero. In contrast to this, $\delta_{\text{min}} \rightarrow 0$ faster for smaller $\text{Re}\ell$.

Moreover, as $\text{Im}\ell$ is growing, the bistability existence region, at first, gets much more narrow for $\delta < \delta_{\text{min}}$ and slightly wider for $\delta > \delta_{\text{min}}$ (see Fig. 3). But then it tends to get wider in both cases. In the limit $\text{Im}\ell \rightarrow \infty$, the region of bistability occupies all the half-plane $b > 0$.

III. DIELECTRIC PERMITTIVITY

The resulting dielectric permittivity of the dielectric doped with two-level atoms can be written as

$$\varepsilon_{\text{total}} = 1 + 4\pi\chi, \quad (7)$$

where $\chi = P/E$ is the susceptibility, P is the total polarization defined as a sum of the linear polarization of the background dielectric (permittivity ε) and the nonlinear polarization $P_{\text{res}} = 2N\mu R$ due to resonant atoms, so that

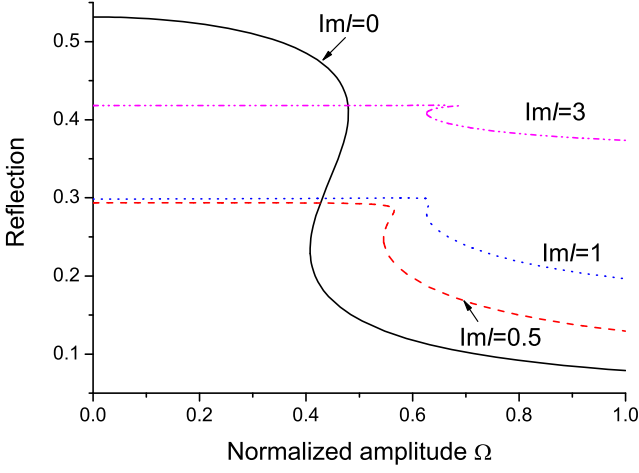


FIG. 6: Dependence of reflection at the vacuum-medium interface on the light amplitude. Calculation parameters are the same as in Fig. 5.

[8]

$$P = \frac{\varepsilon - 1}{4\pi} E + \frac{\varepsilon + 2}{3} P_{\text{res}}. \quad (8)$$

Substituting the stationary expression (4) to the equations (8) and (7) we obtain

$$\varepsilon_{\text{total}} = \varepsilon + \frac{3i\ell^2 bW}{i(\delta - \ell bW) - 1}. \quad (9)$$

It is worth to recall that $\ell = (\varepsilon + 2)/3$.

Expression (9) with cubic equation (5) defines the bistable behavior of the dielectric permittivity (Fig. 5). This bistability, in accordance with the results of the previous section, is getting more and more narrow at first and then wider, as the imaginary part of local-field enhancement factor is growing. Moreover, it is seen that the real part of $\varepsilon_{\text{total}}$ appears to be negative at low-intensive branch of the bistability. This negativity corresponds to the well-studied effect of the internal coherent reflection [9–11]. This phenomenon can be obtained in pure form if the imaginary part of $\varepsilon_{\text{total}}$ is absent. If it were so, then the condition $\text{Re}\varepsilon_{\text{total}} < 0$ would lead to the unit value of the reflection coefficient, i.e. $|(\sqrt{\varepsilon_{\text{total}}} - 1)/(\sqrt{\varepsilon_{\text{total}}} + 1)|^2 = 1$. However, as Fig. 5(b) shows, $\text{Im}\varepsilon_{\text{total}}$ increases for greater $\text{Im}\ell$. As a result, the reflection for large $\text{Im}\varepsilon_{\text{total}}$ is far from unity and it only slightly differs for both branches of the bistability (see Fig. 6). The difference between the branches tends to decrease for $\text{Re}\varepsilon_{\text{total}}$ and $\text{Im}\varepsilon_{\text{total}}$ as well.

IV. NONLOCAL BISTABILITY (PROPAGATION EFFECTS)

Now let us consider the layer of the two-level medium with finite thickness. In this case we have to take into

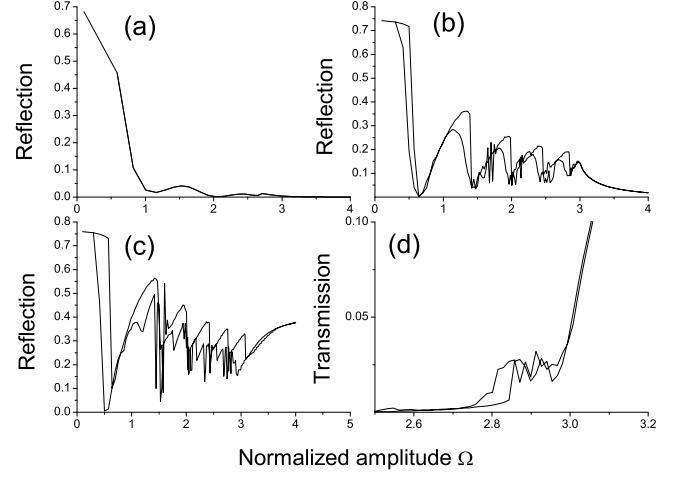


FIG. 7: Dependence of reflection (a, b, c) and transmission (d) on the light amplitude. Calculation parameters: $W_{\text{eq}} = -1$, $\delta = -1$, $b = 4$, $\Gamma = 0.1$, light wavelength $\lambda = 0.5 \mu\text{m}$. The enhancement factor (a) $\ell = 1$, (b, d) $\ell = 2$, (c) $\ell = 3$. The thickness of the layer is $L = 0.5 \mu\text{m}$.

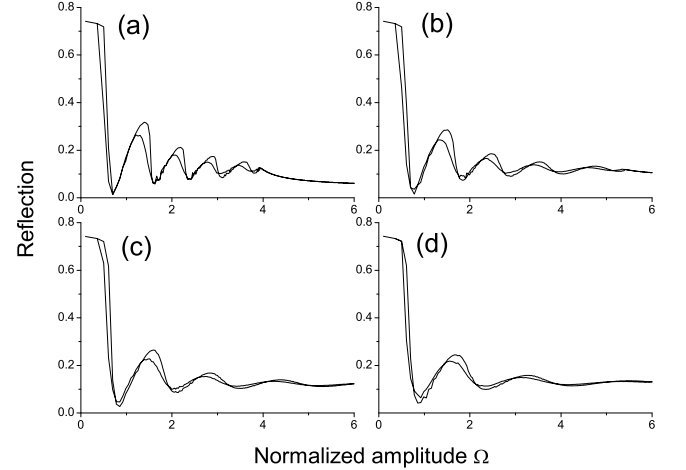


FIG. 8: Dependence of reflection on the light amplitude. The enhancement factor (a) $\ell = 2 + 0.1i$, (b) $\ell = 2 + 0.2i$, (c) $\ell = 2 + 0.3i$, (d) $\ell = 2 + 0.4i$. The other parameters are the same as in Fig. 7.

account the light propagation effects resulting in the so-called nonlocal optical bistability [11]. To analyze propagation of monochromatic light wave in the nonlinear layer, we use the iteration matrix method which was described in detail in Ref. [9]. This method is the modification of the widely known transfer matrix approach. Due to this method, we divide the layer into sublayers with thicknesses much less than the light wavelength so that the properties of each sublayer can be treated as constant. Then, according to initial conditions (refractive index of unexcited medium), we calculate the distribution of light inside the layer and, hence, obtain new values of refractive index of each sublayer. These calculations can be continued many times until, at a certain iteration, the

distribution of light and refractive index take on the stationary form (with a certain accuracy).

The results of calculations of reflection and transmission coefficients are demonstrated in Fig. 7 and 8. In Fig. 7 one can see one more confirmation of the role of the local-field enhancement factor ℓ in bistability appearance. There is also multiplicity of hysteresis loops that can be treated as the main feature of nonlocal bistability [9]. The number of loops depends on the layer thickness and the value of ℓ [compare figures 7(b) and (c)]. Transmission has only one loop corresponding to the most intensive reflection loop [Fig. 7(d)].

Even small absorption of host dielectric significantly changes this bistable response as shown in Fig. 8. As the imaginary part of ℓ grows, the loops tend to stretch along the light amplitude axis. The number of loops and their width are decreasing as well. There is another important consequence of background absorption which is connected with the stability of the effect. As one can see in Fig. 7(b) and (c), reflection at intensities between the loops demonstrate many spikes distributed chaotically. These spikes correspond to the regions of misconvergence of the iteration matrix method so that we have to stop it at a certain iteration and take next intensity value. As opposed to this behavior, the regions of bista-

bility have rather fast convergence. Perhaps, the spikes are due to instability by auto-oscillations mechanism reported in [10, 11]. Figure 8 shows that even low level of absorption leads to elimination of spikes, i.e. response of the system becomes stable. This means that the two-level medium with the background dielectric of quite low absorption can be used to obtain only slightly suppressed multiple bistability without auto-oscillations.

V. CONCLUSION

In this article we have considered semiclassically the general situation of the dense collection of two-level atoms placed into absorbing background dielectric. Of course, the coefficient of dielectric absorption cannot take on arbitrary values. We also do not take into account the problem of medium heating due to radiation absorption. Nevertheless, the analysis carried out above can be useful for proper selection of the media in possible experiments. In particular, the appropriate choice of slightly absorbing (and, hence, slightly heated) background medium can help to obtain the hysteresis response without such instabilities as auto-oscillations.

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